

Poincaré Invariant Three-Body Scattering

Ch. Elster^(a), T. Lin^(a), W.N. Polyzou^(b), W. Glöckle^(c)

(a) Institute of Nuclear and Particle Physics, and Department of Physics and Astronomy,
Ohio University, Athens, OH 45701, USA

(b) Department of Physics and Astronomy, The University of Iowa, Iowa City, IA 52242, USA

(c) Institute for Theoretical Physics II, Ruhr-University Bochum, D-44780 Bochum, Germany

Abstract. Relativistic Faddeev equations for three-body scattering are solved at arbitrary energies in terms of momentum vectors without employing a partial wave decomposition. Relativistic invariance is incorporated within the framework of Poincaré invariant quantum mechanics. Based on a Malfliet-Tjon interaction, observables for elastic and breakup scattering are calculated and compared to non-relativistic ones.

A consistent treatment of intermediate energy reactions requires a Poincaré symmetric quantum theory [1]. In addition, the standard partial wave decomposition, successfully applied below the pion-production threshold [2], is no longer an adequate numerical scheme due to the proliferation of the number of partial waves. Thus, the intermediate energy regime is a new territory for few-body calculations, which waits to be explored.

This work addresses two aspects in this list of challenges: exact Poincaré invariance and calculations using vector variables instead of partial waves. In Ref. [3] the non-relativistic Faddeev equations were solved directly as function of vector variables for scattering at intermediate energies. A key advantage of this formulation lies in its applicability at higher energies, where the number of partial waves proliferates. The Faddeev equation, based on a Poincaré invariant mass operator, has been formulated in detail in [4] and has both kinematical and dynamical differences with respect to the corresponding non-relativistic equation.

The formulation of the theory is given in a representation of Poincaré invariant quantum mechanics where the interactions are invariant with respect to kinematic translations and rotations [5]. The model Hilbert space is a three-nucleon Hilbert space (thus not allowing for absorptive processes). The method used to introduce the NN interactions in the unitary representation of the Poincaré group allows to input of e.g. high-precision NN interactions in a way that reproduces the measured two-body observables. However in this study we use a simpler interaction consisting of a superposition of an attractive and a repulsive Yukawa interaction with parameters chosen such that a bound state at $E_d = -2.23$ MeV is supported [4]. Poincaré invariance and S -matrix cluster properties dictate how

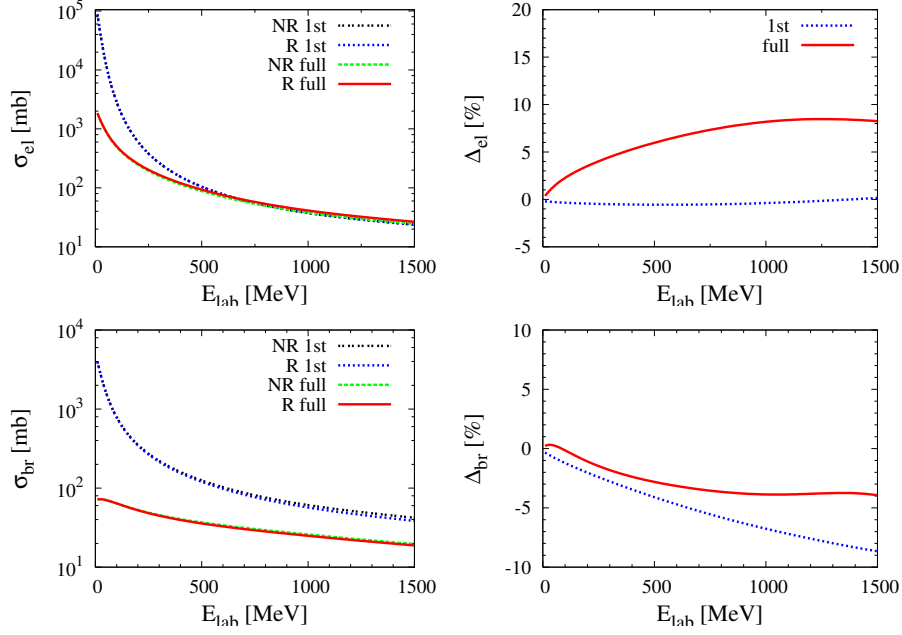


Figure 1. The total elastic c.m. cross section for elastic (top left) and for breakup scattering (bottom left) calculated from a Malfliet-Tjon type potential as function of the projectile kinetic laboratory energy. The labels ‘R’ (‘NR’) stand for relativistic (non-relativistic) calculations. The Faddeev calculations in the first order in t are marked with ‘1st’, the converged full Faddeev calculations with ‘full’. To show the difference, the percentage difference between the relativistic and corresponding non-relativistic calculations are displayed on the right.

the two-body interactions must be embedded in the three-body dynamical generators. Scattering observables are calculated using Faddeev equations formulated with the mass Casimir operator (rest Hamiltonian) constructed from these generators.

To obtain a valid estimate of the size of relativistic effects, it is important that the interactions employed in the relativistic and non-relativistic calculations are phase-shift equivalent. We follow here the suggestion by Coester, Piper, and Serduke (CPS) in constructing a phase equivalent interaction from a non-relativistic 2N interaction [6] by adding the interaction to the square of the mass operator. In this CPS method the relativistic interaction can not be analytically calculated from the non-relativistic one. However, there is a simple analytic connection between the relativistic and non-relativistic two-body t -matrices

$$t_{re}(\mathbf{p}, \mathbf{p}'; 2E_p^{rel}) = \frac{2m}{\sqrt{m^2 + p^2} + \sqrt{m^2 + p'^2}} t_{nr}(\mathbf{p}, \mathbf{p}'; 2E_p^{nr}), \quad (1)$$

where $2E_p^{rel} = 2\sqrt{m^2 + p^2}$ and $2E_p^{nr} = \frac{p^2}{m} + 2m$. This relativistic two-body t -matrix $t_{re}(\mathbf{p}, \mathbf{p}'; 2E_p^{rel})$ is scattering equivalent to the non-relativistic one at the same relative momentum \mathbf{p} [7]. This t -matrix serves then as input to obtain the Poincaré invariant transition amplitude of the 2N subsystem embedded in the three-particle Hilbert space via a first resolvent method as layed out in Ref. [4].

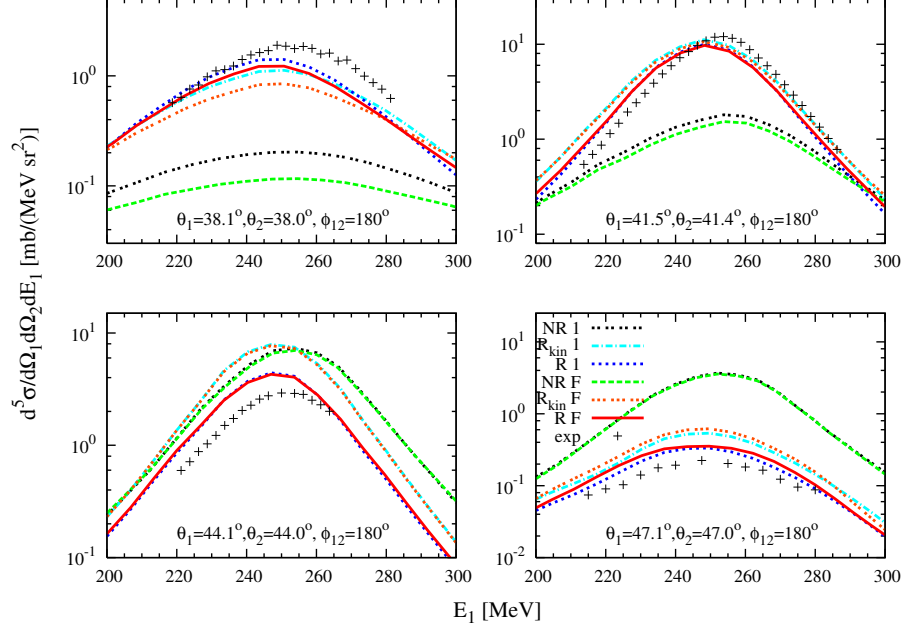


Figure 2. The exclusive differential cross section for the reaction $^2\text{H}(p,2p)n$ at 508 MeV laboratory projectile energy for different proton angle pairs $\theta_1 - \theta_2$ symmetric around the beam axis as function of the laboratory kinetic energy of one of the outgoing protons. The meaning of the curves are the same as in Fig. 1, except that here ‘1’ denotes the 1st order Faddeev calculation, ‘F’ the fully converged one. In the curves labeled R_{kin} only relativistic kinematics is taken into account. The data are taken from Ref. [10].

By construction, differences in the relativistic and non-relativistic calculations first appear in the three-body calculations. Those differences are in the choice of kinematic variables (Jacobi momenta are constructed using Lorentz boosts rather than Galilean boosts) and in the embedding of the two-body interactions in the three-body problem, which is a consequence of the non-linear relation between the two and three-body mass operators. These differences modify the permutation operators and the off-shell properties of the kernel of the Faddeev equations [9].

In Fig. 1 the total cross sections for elastic and breakup cross sections are displayed as function of the projectile kinetic energy up to 1.5 GeV obtained from our fully converged relativistic Faddeev calculation as well as the one obtained from the first-order term, $T^{1st} = tP$, with P being the permutation operator for three identical particles. It is obvious that, especially for energies below 300 MeV, the contribution of rescattering terms is huge. However, for extracting the size of relativistic effect, it is more useful to consider the relative difference between the relativistic and non-relativistic calculations. In first order, there is essentially no effect in the total elastic cross section, which is consistent with the observation that when the relativistic two-body t -matrix is constructed to be phase-shift

equivalent to the non-relativistic one. Making the same comparison with fully converged Faddeev calculations indicates that relativistic effects in the three-body problem increase the total cross section for elastic scattering with increasing energy, whereas it is slightly reduced in the total breakup cross section.

Considering exclusive breakup reactions, differences between a relativistic and non-relativistic calculation can be more pronounced and strongly depend on the specific configuration. Though our two-body force is simple, we compare to a ${}^2\text{H}(\text{p}, 2\text{p})\text{n}$ experiment at 508 MeV [10] to see if our calculation captures essential features of the measurement. Differences in the predictions of our relativistic and non-relativistic calculations are very pronounced at this energy as can be seen in Fig. 2, which shows selected angle pairs $\theta_1 - \theta_2$ from Ref. [10], which are symmetric around the beam axis. The cross section is plotted against the laboratory kinetic energy of one of the outgoing protons. It is interesting to observe that for smaller angle pairs the relativistic cross sections (RF) are considerably larger than the non-relativistic ones (NRF). For larger angle pairs the situation reverses. It is further noteworthy, that in the configurations of Fig. 2, which are close to quasi-free, rescattering effects (or equivalently higher order contributions of the Faddeev multiple scattering series) are very small. To show that peak-positions are given by kinematics, we added a curve labeled ‘ R_{kin} ’ to the figures, which stands for a non-relativistic calculation in which only kinematics and phase space factors are replaced by the relativistic ones. We want to note that the above comparisons do not involve a non-relativistic limit, instead relativistic and non-relativistic three-body calculations with interactions that are fit to the same two-body data are compared. All of the differences are due to the different ways two-body dynamics is incorporated in the three-body problem.

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